Buying the Option to say "No"

Günther Lang

Universidade Nova de Lisboa
Faculdade de Economia*

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Abstract

We analyze a simple model of bilateral bargaining under asymmetric information where the seller of an object can not simply say "no" by default to a buyer who is supposed to make a take-it-or-leave-it offer. Rather, he must acquire this option before the actual bargaining process begins. This choice is observable to the buyer, and hence, the seller's pre–bargaining action might signal private information. We develop a complete characterization of Perfect Bayesian Equilibrium in pure and (strictly) mixed strategies for this game. Then the model is compared to a standard bargaining setting in terms of the realization of welfare enhancing property–right changes.
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*Author's address: Günther Lang, Universidade Nova de Lisboa, Faculdade de Economia, Travessa Estêvão Pinto (Campolide) 1099-032 Lisboa, Portugal.

Introduction

Efficient trade among economic agents requires a well-established property-rights system that protects initial endowments as well as produced goods. Without such an institution, incentives to exert effort are distorted as the total (marginal) gains from individual performance do not fall exclusively to the individual himself. It is frequently argued that the protection of property rights has public good character, and indeed, police protection and criminal investigation are provided on a collective base.

However, more basic instruments of protection against burglary and theft are left to the individual or local level: door locks from family homes to industrial complexes are financed by their owners, electronic video surveillance systems and private security in condominiums and factories are also organized on an individual base, firms with internet presence spend considerable amounts of money in the electronic protection of sensitive data that is only for internal use. Clearly, one can argue that these instruments are rather goods of private benefit, or at most public goods on a restricted local level. Nevertheless, there may well exist significant scale effects in order to justify joint provision of these goods and services.

On the other hand, the magnitude of investment undertaken by an individual party in the protection of its property–rights in a certain object may reveal (or conceal) significant information as to the valuation of the object by this party. Expenses made on protection may therefore be of value in trade relationships under asymmetric information as a signalling device: high valuation types of sellers could use this instrument for the purpose of distinguishing themselves from low valuation types, inducing potentially higher bids from buyers who are supposed to make a (first) offer. Hence, welfare–enhancing exchanges in property–rights may be promoted as compared to a setting where scale–effects would justify joint supply of protective measures.

We will formally analyze the effect of introducing a cost for having the option to say "no" into a traditional model of bilateral bargaining where a buyer makes a take-it-or-leave-it proposal to a seller (see Fudenberg and Tirole (1983), and (1991), Ch. 10, Sec. 2). The following section describes the model, subsequently all equilibria of the game in pure and mixed strategies are examined.

The Model

We consider a traditional one-shot single-offer bargaining game with asymmetric information, with the extension that the seller can only veto the transfer of the object in his possession if he has made an initial investment in the protection of his property–right. The setting where there is no investment stage, and the seller can always say "no" is referred to as the *standard (bargaining) game.*

We assume that for buyer (A) and seller (B) there are two possible types, respectively: $v \in \{\underline{v}, \overline{v}\}$ for A, with a-priori distribution $p(v), p(\underline{v}) + p(\overline{v}) = 1$; $w \in \{\underline{w}, \overline{w}\}$ for B, with a-priori distribution $q(w), q(\underline{w}) + q(\overline{w}) = 1$. The types for both of the players stem from an independent draw. We will examine the two relevant cases: the *no-gap case* with $0 \leq \underline{w} < \underline{v} < \overline{w} < \overline{v}$, and the *gap-case* with $0 \leq \underline{w} < \overline{w} \leq \underline{v} < \overline{v}$. Concerning the amount k necessary for the seller to invest in order to be able to decline the buyer's offer, we assume that it is fixed with $0 \leq \underline{w} < k < \overline{w}$.

The timing of the game is as follows:

(i) Nature decides independently, according to the distributions $p(v)$ and $q(w)$, the types of both players and exclusively reveals to each of them his own type.

(ii) Seller B decides whether or not to invest k. If he does not invest then the object changes hands with a payoff of 0 for the seller and v for the buyer. Otherwise, the game continues with step (iii).

(iii) Buyer A makes an offer c to B.

(iv) Seller B decides whether or not to accept this offer. If he does, then the object changes hands, with payoffs of c for B and $v - c$ for A. If he does not, B stays with the object and a payoff of w, whereas A earns 0.

One could equally well imagine a setting where the seller first invests and then makes an offer to the buyer. This problem then is only of unilateral asymetric information and trivial to solve: the seller invests iff his maximal expected payoff over all his bids exceeds the cost k. In the game analyzed here, however, a true signalling problem arises, in the sense that B's decision to invest may reveal or hide information about his type.

We denote by $\delta_k(w) \in \{0, 1\}$ B's behavorial strategy when asked whether to invest (1) or not (0). This decision may be conditioned on B's type. Analoguously, we denote by $c(v)$ A's bid-strategy. By $a(c|w) \in \{0, 1\}$ we designate B's acceptance (1) or refusal (0) of A's offer. This strategy as well may depend on the history of the game revealed to B. The equilibrium concept applied is that of *perfect Bayesian equilibrium* (see Fudenberg and Tirole (1991), Ch. 8).

As a preliminary result, it is clear that B's type $\overline{w}$ always invests in equilibrium because his valuation of the object exceeds the cost of the investment, $k < \overline{w}$. However, type $\underline{w}$ may invest as well, although his valuation is lower than costs: in doing so he avoids revealing his type, which tendentially increases the expected offer made by the buyer as compared to the case where the latter knew that he faces B's type $\underline{w}$.

In order to show that private provision of property–right protection may well outmatch joint supply, we assume an alternative setting with n sellers (owners) where public provision causes costs of G, independently of n, and where $G/n < k$ (so scale–effects are supposed to have turned public provision into the less costly alternative). This public system is mandatory for all sellers, with equal cost–sharing, and once it is implemented sellers and buyers play the standard game with the sellers' default option to refuse an offer.

No-Gap Case: $\underline{w} < \underline{v} < \overline{w} < \overline{v}$

In what follows we will first analyze existence of equilibrium in pure strategies. Then we look at equilibria in (strictly) mixed strategies.

Theorem 1 (Separating Equilibrium) *Separating equilibria do exist iff*

$$\underline{w} \leq \tilde{w} := \frac{k - p(\overline{v})\overline{w}}{p(\underline{v})}. \tag{1}$$

For all $\breve{w} \leq \tilde{w}$, the following strategies constitute a separating equilibrium: $\delta_k(\underline{w}) = 0, \delta_k(\overline{w}) = 1$, $c(\underline{v}) = \breve{w}$, $c(\overline{v}) = \overline{w}$, and

$$a(c|w) = \begin{cases} 1 & \text{for } c \geq w \\ 0 & \text{for } c < w \end{cases} \tag{2}$$

for all c and w, with beliefs at out-of-equilibrium-path information sets chosen arbitrarily.

Proof: Out-of-equilibrium-path information sets can only be found at the stage where B has to decide acceptance or refusal of A's offer. There, as well as at information sets which are on the equilibrium path at this stage, it is best for B to accept an offer that matches at least w, independently of B's beliefs about A's type ; hence $a(\cdot)$ as given above is optimal.
Given that $\delta_k = 1$ is observed, Bayes' rule requires A to believe in B's type $\overline{w}$. A's type $\overline{v}$ then just offers $\overline{w}$, knowing that this offer will be accepted. A's type $\underline{v}$ could induce $\overline{w}$'s acceptance only by incurring a loss. Since $\breve{w} \leq \tilde{w} < \overline{w}$, he avoids just this. Therefore, $c(\overline{v}) = \overline{w}$ and $c(\underline{v}) = \breve{w}$, respectively, are optimal for the beliefs implied by the equilibrium strategies.
B's type $\overline{w}$ always invests: Without doing so his payoff would be zero since he could not recuse A's proposal, but having invested offers the opportunity to say no and guarantees him a payoff of $\overline{w} - k > 0$. $\breve{w} \leq \tilde{w}$ (as given in (1)) implies that $p(\underline{v})\breve{w} + p(\overline{v})\overline{w} - k \leq 0$, and so $\underline{w}$ finds it optimal not to invest.
On the other hand, if (1) is not fulfilled, i.e. $\underline{w} > \tilde{w}$, then $\underline{w}$ strictly prefers to invest for any $\breve{w} > 0$ since his reservation payoff is $\underline{w}$ anyway. Hence, (1) is also necessary for the existence of separating equilibria. ■

Relationship (1) is equivalent to $p(\overline{v}) \leq \frac{k-\underline{w}}{\overline{w}-\underline{w}}$. The interpretation for the existence of a separating equilibrium is that since the probability of meeting A's type $\overline{v}$ is relatively low — and this type is the only who potentially offers $\overline{w}$ — B's type $\underline{w}$ does not find it worthwhile to invest.

Theorem 2 (Pooling Equilibrium) *A pooling equilibrium does exist iff*

$$\underline{w} \geq \tilde{w} := \frac{k - p(\overline{v})\overline{w}}{p(\underline{v})} \tag{3}$$

and

$$\underline{w} \geq \hat{w} := \overline{v} - \frac{\overline{v} - \overline{w}}{q(\underline{w})}. \tag{4}$$

The following strategies constitute this equilibrium: $\delta_k(\underline{w}) = 1, \delta_k(\overline{w}) = 1$, $c(\underline{v}) = \underline{w}$, $c(\overline{v}) = \overline{w}$, *and* $a(c|w)$ *as in (2). Beliefs at out-of-equilibrium-path information sets can be chosen arbitrarily.*

Proof: Out-of-equilibrium-path information sets can be found at the stage where B has to decide acceptance or refusal of A's offer. By the same argument as in the proof of Theorem 1, $a(\cdot)$ is optimal off as well as on the equilibrium path.
We have $c(\underline{v}) = \underline{w}$, since offering $\overline{w}$ would leave A's type $\underline{v}$ with a certain loss. Offering $\underline{w}$, however, attracts B's type $\underline{w}$, and only this type. This event occurs with probability $q(\underline{w})$, leaving type $\underline{v}$ therefore with an expected payoff of $q(\underline{w})(\underline{v} - \underline{w}) > 0$.
A's type $\overline{v}$ offers $c(\overline{v}) = \overline{w}$ *iff* $\overline{v} - \overline{w} \geq q(\underline{w})(\overline{v} - \underline{w})$, which is equivalent to (4). Otherwise he would offer only $\underline{w}$, in which case both types of A would bid only the low valuation. Then, however, $\underline{w}$ would not invest. Hence, (4) is necessary for the existence of a pooling equilibrium.
B's type $\overline{w}$ invests, by the same (obvious) argument as in Theorem 1. Whether $\underline{w}$ invests, given that (4) is fulfilled, depends: he invests *iff* $p(\underline{v})(\underline{w} - k) + p(\overline{v})(\overline{w} - k) \geq 0$, which is equivalent to (3).
Hence (3) and (4) are necessary and sufficient for the existence of a pooling equilibrium. ■

In a pooling equilibrium, A's type $\overline{v}$ must find it worthwhile to bid $\overline{w}$. This is the case if the probability of meeting B's type $\underline{w}$ is not too high; indeed (4) is equivalent to $q(\underline{w}) \leq \frac{\overline{v}-\overline{w}}{\overline{v}-\underline{w}}$. B's type $\underline{w}$ then invests as well if the probability of meeting A's type $\overline{v}$ is sufficiently high, which is equivalent to (3).

Existence of either type of equilibrium therefore depends on the relationship between $\hat{w}$ and $\tilde{w}$:

$\hat{\mathbf{w}} \leq \tilde{\mathbf{w}}$: Separating equilibria do exist *iff* $\underline{w} \leq \tilde{w}$. Pooling equilibria do exist *iff* $\underline{w} \geq \tilde{w}$. For $\underline{w} = \tilde{w}$ separating and pooling equilibria coexist with an expected payoff for B's type $\underline{w}$ of zero in both cases.

$\hat{\mathbf{w}} > \tilde{\mathbf{w}}$: Separating equilibria do exist *iff* $\underline{w} \leq \tilde{w}$. Pooling equilibria do exist *iff* $\underline{w} \geq \hat{w}$. For $\tilde{w} < \underline{w} < \hat{w}$ neither pooling nor separating equilibria do exist: a) If A's type $\overline{v}$ would offer $\overline{w}$, then B's type $\underline{w}$ where induced to invest as well. However, $\underline{w}$'s frequency is too high in order to make it worthwhile for $\overline{v}$ to do so, and b) the frequency of A's type $\overline{v}$, who in a separating equilibrium is supposed to offer $\overline{w}$, is too high for B's type $\underline{w}$ not to invest.

In the following we consider all cases of equilibria in (strictly) mixed strategies that exist in addition to the pure-strategy equilibria already discussed.

Theorem 3 (Mixed-Strategy Equilibria) *In the no-gap case the following equilibria in (strictly) mixed strategies do exist:*

(i) For

$$\underline{w} < \tilde{w} := \frac{k - p(\overline{v})\overline{w}}{p(\underline{v})} \tag{5}$$

the only mixed-strategy equilibria consist of the (pure) strategies from the separating equilibrium of Theorem 1, with the exception that A's type $\underline{v}$ mixes his bid $c(\underline{v})$ on the interval $[0, \tilde{w}]$.

(ii) For

$$\underline{w} = \tilde{w} := \frac{k - p(\overline{v})\overline{w}}{p(\underline{v})} \tag{6}$$

the following strategies constitute the set of all hybrid equilibria: $(prob(\delta_k(\underline{w}) = 1) = \gamma, prob(\delta_k(\underline{w}) = 0) = 1 - \gamma), \delta_k(\overline{w}) = 1,\ c(\underline{v}) = \underline{w}$ (for $\gamma = 0$ any mixed strategy on $[0, \tilde{w}]$ as in (i)), $c(\overline{v}) = \overline{w}$, and $a(c|w)$ as in (2), with $\gamma \leq \min(\gamma^*, 1)$, where $\gamma^* > 0$ is the (unique) solution to

$$\frac{\gamma^* q(\underline{w})}{\gamma^* q(\underline{w}) + q(\overline{w})}(\overline{v} - \underline{w}) = \overline{v} - \overline{w}. \tag{7}$$

(iii) For

$$\underline{w} > \tilde{w} := \frac{k - p(\overline{v})\overline{w}}{p(\underline{v})} \tag{8}$$

then:

a) For $q(\underline{w})(\overline{v} - \underline{w}) = \overline{v} - \overline{w}$ (i.e. $\underline{w} = \hat{w}$), the pooling equilibrium from Theorem 2 survives with the modification that A's type $\overline{v}$ mixes: $(prob(c(\overline{v}) = \underline{w}) = \overline{\alpha}, prob(c(\overline{v}) = \overline{w}) = 1 - \overline{\alpha})$, with $\overline{\alpha} \leq \overline{\alpha}^*$, where $\overline{\alpha}^* \in (0,1)$ is the (unique) solution to

$$p(\underline{v})(\underline{w} - k) + p(\overline{v})[\overline{\alpha}^*(\underline{w} - k) + (1 - \overline{\alpha}^*)(\overline{w} - k)] = 0. \tag{9}$$

b) For $q(\underline{w})(\overline{v}-\underline{w}) > \overline{v}-\overline{w}$ (i.e. $\underline{w} < \hat{w}$), then the following strategies constitute the set of all hybrid equilibria:
$(prob(\delta_k(\underline{w}) = 1) = \gamma^*, prob(\delta_k(\underline{w}) = 0) = 1-\gamma^*), \delta_k(\overline{w}) = 1$, $c(\underline{v}) = \underline{w}$, $(prob(c(\overline{v}) = \underline{w}) = \overline{\alpha}^*, prob(c(\overline{v}) = \underline{w}) = 1-\overline{\alpha}^*)$, and $a(c|w)$ as in (2), with $\overline{\alpha}^*$ and γ^* as defined in (ii) and (iii) a), respectively.

In all cases, beliefs at out-of-equilibrium-path information sets can be chosen arbitrarily.

Proof: (i) From Theorem 1 we know that B's type $\underline{w}$ finds it optimal not to invest for any $c(\underline{v}) \leq \tilde{w}$. Since $\tilde{w} < \overline{w}$, B's type $\overline{w}$ rejects such an offer anyway. Mixing on the interval $[0, \tilde{w}]$ then has the same effect. The mixed-strategy equilibrium is payoff-equivalent to the separating equilibrium.
It remains to show that there are no other mixed-strategy equilibria. First of all, in order to guarantee existence of a best response it is required to assume that, on the equilibrium path, both types of B accept A's offer if indifferent between accepting and rejecting. Then (5) implies that $\underline{w}$ would not invest even if A's type $\overline{v}$ offered $\overline{w}$, and, on the other hand, if he invests then A's type $\underline{v}$ would never offer more than $\underline{w}$. Therefore, $\underline{w}$ never invests in equilibrium, and so only the separating equilibrium from Theorem 1, with mixing as just described, survives.
(ii) Given that (6) is satisfied, B's type $\underline{w}$ is indifferent between investing and not investing, provided that $c(\underline{v}) = \underline{w}$ and $c(\overline{v}) = \overline{w}$; and therefore he may mix. The lhs of (7) gives the expected payoff of A's type $\overline{v}$ if he bids $\underline{w}$, whereby the first term expresses the Bayesian updated probability of type $\underline{w}$ given this type's mixed strategy. The rhs of (7) is type $\overline{v}$'s certain payoff if bidding $\overline{w}$. There trivially exists a unique solution γ^* to (7) that is positive but not necessarily below or equal to one. If B's type $\underline{w}$ mixes with probability $\gamma \leq \min(\gamma^*, 1)$ then the lhs of (7) is lower than or equal to the rhs, and so it is optimal for type $\overline{v}$ to offer $c(\overline{v}) = \overline{w}$. If A's type $\underline{v}$ mixes on $[0, \tilde{w}]$ then $\gamma = 0$ is optimal for B's type $\underline{w}$, and moreover, if any strategy from $[0, \tilde{w})$ is played with strictly positive probability then only $\gamma = 0$ is optimal.
(iii) a) and b) Given that (8) holds, B's type $\underline{w}$ is indifferent between investing and not investing if (9) is satisfied, and he strictly prefers to invest if the lhs of (9) is strictly positive. $\overline{\alpha}^* = \frac{p(\overline{v})(\overline{w}-k)+p(\underline{v})(\underline{w}-k)}{p(\overline{v})(\overline{w}-\underline{w})}$ solves (9) and lies in the interval $(0,1)$, whereas $\overline{\alpha} < \overline{\alpha}^*$ implies that the lhs is strictly positive. Hence, if B's type $\underline{w}$ invest with probability one, then $q(\underline{w})(\overline{v}-\underline{w}) = \overline{v}-\overline{w}$ implies that A's type $\overline{v}$ is indifferent between bidding $\underline{w}$ and $\overline{w}$. If $q(\underline{w})(\overline{v}-\underline{w}) > \overline{v}-\overline{w}$ then the γ^* that solves (7) guarantees this indifference.
With the same argument as in Theorems 1 and 2, beliefs at the out-of-equilibrium-path information sets are immaterial. ■

It is important to note that for $\tilde{w} < \underline{w} < \hat{w}$, in which case neither separating nor pooling equilibria do exist, at least equilibrium in mixed strategies can be guaranteed by part (iii) b) of the theorem.

Finally, we compare the expected efficiency losses due to assymetric information in the present model to those in the joint–provision scenario with the guaranteed option to say "no" after the mandatory per–head share G/n is paid (and after which seller and buyer play the standard model). Inefficiency in both models arises in case that a beneficial trade between A and B does not occur. For the no-gap case, in equilibrium, trade between B's type $\underline{w}$ and any of A's types always is beneficial and in fact takes place. However, in the standard model, B's type $\overline{w}$ cannot realize the beneficial trade with A's type $\overline{v}$ if the probability of B's type $\underline{w}$ is too high, in particular if $q(\underline{w}) > \frac{\overline{v}-\overline{w}}{\overline{v}-\underline{w}}$. In the model discussed here, this case is consistent only with the separating equilibrium and the mixed-strategy equilibria (i) and (ii). In any of these equilibria, all welfare increasing transfers in property-rights do take place — although not always voluntarily — but social cost consist of B's type $\underline{w}$'s and/or $\overline{w}$'s investment k. Therefore, expected efficiency losses are $q(\overline{w})k$ in the separating equilibrium, as well as in the mixed-strategy equilibrium (i), and they amount to $[q(\underline{w})\gamma + q(\overline{w})]k$ in case of mixed-strategy equilibrium (ii). In the standard model they are $G/n + p(\overline{v})q(\overline{w})(\overline{v} - \overline{w})$. Hence, having investment in property-rights protection as a signalling device pays off iff $k < \frac{G}{nq(\overline{w})} + p(\overline{v})(\overline{v} - \overline{w})$ in the first two cases, and $k < \frac{G/n+p(\overline{v})q(\overline{w})(\overline{v}-\overline{w})}{q(\underline{w})\gamma+q(\overline{w})}$ in the last case. On the other hand, if $q(\underline{w}) < \frac{\overline{v}-\overline{w}}{\overline{v}-\underline{w}}$, all efficient trades in the standard model take place, in the model discussed here, however, there are always social costs in terms of at least type $\overline{w}$'s investment k.

Gap Case: $\underline{w} < \overline{w} \leq \underline{v} < \overline{v}$

Theorem 4 (Pure-Strategy Equilibria) *In the gap case*

a) no separating equilibria do exist, and

b) a pooling equilibrium does exist iff

(i) $\underline{w} \geq \hat{w}(\underline{v})$, where

$$\hat{w}(v) := v - \frac{v - \overline{w}}{q(\underline{w})}, \tag{10}$$

or

(ii) $(\hat{w}(\overline{v}) \leq \underline{w} < \hat{w}(\underline{v})$ and $\underline{w} \geq \tilde{w}$ ($\tilde{w}$ given by (1))).

The following strategies constitute this equilibrium: $\delta_k(\underline{w}) = 1, \delta_k(\overline{w}) = 1$, $c(\underline{v}) = \overline{w}$ (case (i)), $c(\underline{v}) = \underline{w}$ (case (ii)), $c(\overline{v}) = \overline{w}$, and $a(c|w)$ as in (2). Beliefs at out-of-equilibrium-path information sets can be chosen arbitrarily.

Proof: $\hat{w}(v)$ as given in (10) makes type v indifferent between offering $\overline{w}$ and $\hat{w}(v) < \overline{w}$. Note that $\hat{w}(\overline{v}) < \hat{w}(\underline{v})$. Then, if $\underline{w} < \hat{w}(\overline{v})$, none of A's types offers $\overline{w}$. Hence, B's type $\underline{w}$ does not invest.
If $\hat{w}(\overline{v}) \leq \underline{w} < \hat{w}(\underline{v})$, then $c(\underline{v}) = \underline{w}$ and $c(\overline{v}) = \overline{w}$. Also type $\underline{w}$ then finds it optimal to invest *iff* $\underline{w} \geq \tilde{w}$.
If $\underline{w} \geq \hat{w}(\underline{v})$, then both types of A bid the high valuation $\overline{w}$. In this case, both types of B find it optimal to invest.
By the same argument as above, beliefs at out-of-equilibrium information sets where B has to decide acceptance or refusal are irrelevant, and $a(\cdot)$ is optimal. ■

Also in the gap case we now investigate equilibrium in (strictly) mixed strategies.

Theorem 5 (Mixed-Strategy Equilibria) *In the gap case the following equilibria in (strictly) mixed strategies do exist:*

(i) For $\underline{w} = \hat{w}(\underline{v})$*:* $\delta_k(\underline{w}) = 1$, $\delta_k(\overline{w}) = 1$, $prob(c(\underline{v}) = \underline{w}) = \underline{\alpha}, prob(c(\underline{v}) = \overline{w}) = 1 - \underline{\alpha})$*, with* $\underline{\alpha} \leq \min(\underline{\alpha}^*, 1)$, $c(\overline{v}) = \overline{w}$*, and* $a(c|w)$ *as in (2).*

(ii) For $\hat{w}(\overline{v}) < \underline{w} < \hat{w}(\underline{v})$*:*

- *a) For* $\underline{w} = \tilde{w}$*:* $(prob(\delta_k(\underline{w}) = 1) = \gamma, prob(\delta_k(\underline{w}) = 0) = 1 - \gamma)$*, with* $1 > \gamma \geq \gamma^*(\underline{v})$, $c(\underline{v}) = \underline{w}$, $c(\overline{v}) = \overline{w}$*, and* $a(c|w)$ *as in (2).*
- *b) For* $\underline{w} < \tilde{w}$*:* $(prob(\delta_k(\underline{w}) = 1) = \gamma^*(\underline{v}), prob(\delta_k(\underline{w}) = 0) = 1 - \gamma^*(\underline{v}))$, $(prob(c(\underline{v}) = \underline{w}) = \underline{\alpha}^*, prob(c(\underline{v}) = \overline{w}) = 1 - \underline{\alpha}^*)$, $c(\overline{v}) = \overline{w}$*, and* $a(c|w)$ *as in (2).*

(iii) For $\underline{w} = \hat{w}(\overline{v})$*:*

- *a) For* $\underline{w} > \tilde{w}$*:* $\delta_k(\underline{w}) = 1$, $\delta_k(\overline{w}) = 1$, $c(\underline{v}) = \underline{w}$, $(prob(c(\overline{v}) = \underline{w}) = \overline{\alpha}, prob(c(\overline{v}) = \overline{w}) = 1 - \overline{\alpha})$*, with* $\overline{\alpha} \leq \overline{\alpha}^*$*, and* $a(c|w)$ *as in (2).*
- *b) For* $\underline{w} = \tilde{w}$*:* $(prob(\delta_k(\underline{w}) = 1) = \gamma, prob(\delta_k(\underline{w}) = 0) = 1 - \gamma)$*, with* $1 > \gamma \geq \gamma^*(\underline{v})$, $c(\underline{v}) = \underline{w}$, $c(\overline{v}) = \overline{w}$*, and* $a(c|w)$ *as in (2).*
- *c) For* $\underline{w} < \tilde{w}$*:* $(prob(\delta_k(\underline{w}) = 1) = \gamma^*(\underline{v}), prob(\delta_k(\underline{w}) = 0) = 1 - \gamma^*(\underline{v}))$, $(prob(c(\underline{v}) = \underline{w}) = \underline{\alpha}^*, prob(c(\underline{v}) = \overline{w}) = 1 - \underline{\alpha}^*)$, $c(\overline{v}) = \overline{w}$*, and* $a(c|w)$ *as in (2).*

iv) For $\underline{w} < \hat{w}(\overline{v}) < \hat{w}(\underline{v})$*:*

- *a) For* $\underline{w} > \tilde{w}$*:* $(prob(\delta_k(\underline{w}) = 1) = \gamma^*(\overline{v}), prob(\delta_k(\underline{w}) = 0) = 1 - \gamma^*(\overline{v}))$, $c(\underline{v}) = \underline{w}$, $(prob(c(\overline{v}) = \underline{w}) = \overline{\alpha}^*, prob(c(\overline{v}) = \overline{w}) = 1 - \overline{\alpha}^*)$*, and* $a(c|w)$ *as in (2).*
- *b) For* $\underline{w} = \tilde{w}$*:* $(prob(\delta_k(\underline{w}) = 1) = \gamma, prob(\delta_k(\underline{w}) = 0) = 1 - \gamma)$*, with* $\gamma^*(\underline{v}) \leq \gamma \leq \gamma^*(\overline{v})$, $c(\underline{v}) = \underline{w}$, $c(\overline{v}) = \overline{w}$*, and* $a(c|w)$ *as in (2).*

c) For $\underline{w} < \tilde{w}$: $(prob(\delta_k(\underline{w}) = 1) = \gamma^*(\underline{v}), prob(\delta_k(\underline{w}) = 0) = 1 - \gamma^*(\underline{v}))$, $(prob(c(\underline{v}) = \underline{w}) = \underline{\alpha}^*, prob(c(\underline{v}) = \overline{w}) = 1 - \underline{\alpha}^*)$, $c(\overline{v}) = \overline{w}$, and $a(c|w)$ as in (2).

Thereby, $\gamma^*(v)$ is the (unique) solution to

$$\frac{\gamma^*(v)q(\underline{w})}{\gamma^*(v)q(\underline{w}) + q(\overline{w})}(v - \underline{w}) = v - \overline{w}, \tag{11}$$

$\overline{\alpha^*}$ as given in (9), and $\underline{\alpha^*}$ is the (unique) solution to

$$p(\underline{v})[\underline{\alpha}^*(\underline{w} - k) + (1 - \underline{\alpha}^*)(\overline{w} - k)] + p(\overline{v})(\overline{w} - k) = 0. \tag{12}$$

In all cases, beliefs at out-of-equilibrium-path information sets can be chosen arbitrarily.

Proof: Given A's type $v \in \{\underline{v}, \overline{v}\}$, he is indifferent between offering $\underline{w}$ and $\overline{w}$ iff (11) holds. There is a unique solution to (11), and $\gamma < \gamma^*(v)$ $(\gamma > \gamma^*(v))$ implies that type v strictly prefers bidding $\overline{w}$ $(\underline{w})$. Also, $\gamma^*(\underline{v}) < \gamma^*(\overline{v})$, i.e. A's high-valuation type $\overline{v}$ is willing to offer $\overline{w}$ at a higher probability of facing B's type $\underline{w}$ as compared to his low-valuation type $\underline{v}$.
$\overline{\alpha^*}$ as given in (9) marks B's type $\underline{w}$'s indifference between investing or not, given that A's type $\underline{v}$ offers $\underline{w}$ and type $\overline{v}$ mixes on $\{\underline{w}, \overline{w}\}$. Consequently, type $\underline{w}$ strictly prefers to invest (not to invest) iff $\overline{\alpha} < \overline{\alpha}^*$ $(\overline{\alpha} > \overline{\alpha}^*)$.
Analoguously, $\underline{\alpha^*}$ as given in (12) marks B's type $\underline{w}$'s indifference between investing or not, given that A's type $\overline{v}$ offers $\overline{w}$ and type $\underline{v}$ mixes on $\{\underline{w}, \overline{w}\}$. Note that $\underline{\alpha^*}$ is strictly positive and it may be greater than one. Type $\underline{w}$ then strictly prefers to invest (not to invest) iff $\underline{\alpha} < \min(\underline{\alpha}^*, 1)$ $(\underline{\alpha} > \min(\underline{\alpha}^*,1))$.
Therefore, if $\underline{w} > \hat{w}(\underline{v})$, both types of A offer $\overline{w}$ even if both types of B invest, and so there only exists the pooling equilibrium from Theorem 4.
With this knowledge we can now prove claims (i) to (iv).
(i) $\underline{w} = \hat{w}(\underline{v})$ implies that A's type $\overline{v}$ offers $\overline{w}$ and type $\underline{v}$ is indifferent between $\underline{v}$ and $\overline{v}$, given that B's types pool. If type $\underline{v}$ then mixes with $\underline{\alpha} \le \min(\underline{\alpha}^*, 1)$ it is optimal for B's type $\underline{w}$ to invest. On the other hand, if the latter would invest with probability lower than one, then both of A's types should offer $\overline{w}$, inducing him to invest with probability one.
(ii) a) Given B's type $\underline{w}$'s mixing with $1 > \gamma \ge \gamma^*(\underline{v})$, using (11) we see that $c(\underline{v}) = \underline{w}$ and $c(\overline{v}) = \overline{w}$ are indeed optimal. Since $\underline{w} = \tilde{w}$, B's type $\underline{w}$ then indeed is indifferent. On the other hand, if the latter would invest with probability lower than $\gamma^*(\underline{v})$, then both of A's types should offer $\overline{w}$, inducing him to invest with probability one.
(ii) b) Given B's type $\underline{w}$'s mixing with $\gamma^*(\underline{v})$, $c(\overline{v}) = \overline{w}$ is uniquely optimal, whereas A's type $\underline{v}$ is indifferent. Given that the latter mixes with $\underline{\alpha}^*$, which is indeed lower than one because of $\underline{w} < \tilde{w}$, B's type $\underline{w}$ is indifferent between investing or not. If type $\underline{w}$ mixed with $\gamma < \gamma^*(\underline{v})$, then both types of A

would offer $\overline{w}$, inducing pooling. If he used $\gamma > \gamma^*(\underline{v})$, then $c(\underline{v}) = \underline{w}$ and $c(\overline{v}) = \overline{w}$ were optimal, inducing type $\underline{w}$ not to invest because of $\underline{w} < \tilde{w}$.
(iii) a) Given that both types of B pool, $\underline{w} = \hat{w}(\overline{v})$ implies that type $\underline{v}$ strictly prefers offering $\underline{w}$, whereas type $\overline{v}$ is indifferent. By (9), B's type $\underline{w}$ then finds it optimal to invest for $\overline{\alpha} \leq \overline{\alpha}^*$. If type $\underline{w}$ did not invest with probability one, then the optimal strategies for A's types would satisfy $c(\underline{v}) \geq \underline{w}$ and $c(\overline{v}) = \overline{w}$, inducing him to invest because of $\underline{w} > \tilde{w}$.
(iii) b) For $1 > \gamma \geq \gamma^*(\underline{v})$, $c(\underline{v}) = \underline{w}$ and $c(\overline{v}) = \overline{w}$ are optimal. Since $\underline{w} = \tilde{w}$, type $\underline{w}$ is indifferent between investing and not doing so. $\gamma = 1$ implies the pure-strategy pooling equilibrium from Theorem 4. $\gamma < \gamma^*(\underline{v})$ would imply $c(\underline{v}) = c(\overline{v}) = \overline{w}$, making it uniquely optimal for type $\underline{w}$ to invest.
(iii) c) Given $\underline{w}$'s mixing with $\gamma^*(\underline{v})$, type $\underline{v}$ is indifferent, whereas type $\overline{v}$ strictly prefers offering $\overline{w}$. The former's mixing with $\underline{\alpha}^*$ then leaves B's type $\underline{w}$ indifferent whether to invest or not. For $\gamma < \gamma^*(\underline{v})$, both types of A would offer $\overline{w}$, inducing pooling of B's types. $\gamma > \gamma^*(\underline{v})$ then makes $c(\underline{v}) = \underline{w}$ uniquely optimal, and because of $\underline{w} < \tilde{w}$, type $\underline{w}$ would not invest.
(iv) a) Given $\underline{w}$'s mixing with $\gamma^*(\overline{v})$, $c(\underline{v}) = \underline{w}$ is unique optimal choice of $\underline{v}$, whereas $\overline{v}$ is indifferent. Hence, he may mix with $\overline{\alpha}^*$. This, in turn, leaves type $\overline{w}$ indifferent between investing and not investing. $\gamma < \gamma^*(\overline{v})$ implies $c(\underline{v}) = \underline{w}$ or $\overline{w}$ and $c(\overline{v}) = \overline{w}$, making type $\underline{w}$ invest because of $\underline{w} > \tilde{w}$. On the other hand, $\gamma > \gamma^*(\overline{v})$ implies $c(\underline{v}) = c(\overline{v}) = \underline{w}$, so that type $\underline{w}$ would not invest.
(iv) b) For $\gamma \in [\gamma^*(\underline{v}), \gamma^*(\overline{v})]$, $c(\underline{v}) = \underline{w}$ and $c(\overline{v}) = \overline{w}$ are optimal, and because of $\underline{w} = \tilde{w}$, B's type $\underline{w}$ then is indifferent between investing or not. $\gamma < \gamma^*(\underline{v})$ implies $c(\underline{v}) = c(\overline{v}) = \overline{w}$, making it worthwhile for type $\underline{w}$ to invest. $\gamma > \gamma^*(\underline{v})$ induces $c(\underline{v}) = c(\overline{v}) = \underline{w}$, which makes type $\underline{w}$ not to invest.
(iv) c) Given that B's type $\underline{w}$ mixes with $\gamma^*(\underline{v})$, A's type $\underline{v}$ is indifferent between offering $\underline{w}$ and $\overline{w}$, whereas type $\overline{v}$ strictly prefers bidding $\overline{w}$. Given the former's mixing with $\underline{\alpha}^*$, B's type $\underline{w}$ is just indifferent. $\gamma < \gamma^*(\underline{v})$ implies $c(\underline{v}) = c(\overline{v}) = \overline{w}$, which makes it worthwhile for type $\underline{w}$ to invest. $\gamma > \gamma^*(\underline{v})$ implies $c(\underline{v}) = \underline{w}$ and $c(\overline{v}) \leq \overline{w}$, which makes it unattractive for type $\underline{w}$ to invest because of $\underline{w} < \tilde{w}$.
Finally, $a(c|w)$ as in (2) is optimal for both of B's types, independently of beliefs held about A's type. ■

Also in the gap-case we examine under which conditions the investment in property rights protection as a signalling device can improve welfare as compared to the joint– supply/ standard model. In the latter, beneficial trade now may not take place between $\overline{w}$ and either $\underline{v}$ or $\overline{v}$, depending on whether $q(\underline{w}) > h(v) := \frac{v-\overline{w}}{v-\underline{w}}$, i.e. on whether B's type $\underline{w}$'s probability is too high. Note that $h(\underline{v}) < h(\overline{v})$. Three cases are to be distinguished.

$\mathbf{q(\underline{w}) > h(\overline{v}) > h(\underline{v})}$: Trade between $\overline{w}$ and neither $\underline{v}$ nor $\overline{v}$ takes place in the standard model, causing social costs of $G/n + q(\overline{w})[p(\underline{v})(\underline{v} - \overline{w}) + p(\overline{v})(\overline{v} - \overline{w})]$. In our signalling model, this case is consistent only with mixed-strategy equilibrium iv). Given the cost of investment k, plus costs in terms of non-realized beneficial trades, in sub-cases a) – c) of iv) the possibility of signalling provides higher welfare as compared to the standard model iff

$$k < \begin{cases} \dfrac{q(\overline{w})p(\overline{v})(1-\overline{\alpha}^*)(\overline{v}-\overline{w}) + G/n}{\gamma^*(\overline{v})q(\underline{w}) + q(\overline{w})} & \text{sub-case a)} \\ \dfrac{q(\overline{w})p(\overline{v})(\overline{v}-\overline{w}) + G/n}{\gamma q(\underline{w}) + q(\overline{w})} & \text{sub-case b)} \\ \dfrac{q(\overline{w})[p(\overline{v})(\overline{v}-\overline{w}) + p(\underline{v})(\underline{v}-\overline{w})(1-\underline{\alpha}^*)] + G/n}{\gamma^*(\underline{v})q(\underline{w}) + q(\overline{w})} & \text{sub-case c)} \end{cases} \tag{13}$$

$\mathbf{h(\underline{v}) < q(\underline{w}) < h(\overline{v})}$: In this case, the standard model only precludes trade between $\overline{w}$ and $\underline{v}$, causing social costs of $p(\underline{v})q(\overline{w})(\underline{v} - \overline{w}) + G/n$. This constellation is consistent only with pooling equilibrium (ii) and mixed-strategy equilibrium (ii). In the first case, also the signalling model precludes trade between $\overline{w}$ and $\underline{v}$, but it causes the cost of investment k. Hence, welfare as compared to the standard model is lower since $k > G/n$. The same applies for mixed-strategy equilibrium (ii) sub-case a). In sub-case b), there is trade between $\overline{w}$ and $\underline{v}$ with probability $1 - \underline{\alpha}^*$; hence the signalling framework provides higher welfare iff $k < \frac{q(\overline{w})p(\underline{v})(1-\underline{\alpha}^*)(\underline{v}-\overline{w})+G/n}{\gamma^*(\underline{v})q(\underline{w})+q(\overline{w})}$.

$\mathbf{q(\underline{w}) < h(\underline{v}) < h(\overline{v})}$: All efficient trades take place in the standard model. This case corresponds to pooling equilibrium (i), where they are realized as well, at social costs of k, however. Hence, since $k > G/n$, the signalling model displays lower social efficiency.

Conclusion

We have analyzed a simple model of bargaining, modified by the feature that the option to decline an offer must be acquired in advance. In the symmetric information case, only high–valuation sellers would buy this option. However, if the sellers' type is private information, then also low–valuation sellers may undergo this costly investment in order to avoid revealing their type. In the no–gap case, this just happens if the ex-ante probability of the low–valuation type seller is sufficiently low, and that of a high–valuation type buyer is sufficiently high. In the gap case, a separating equilibrium does not exist at all because both types of buyers are supposed to make the same (high) offer. On the other hand, also a pooling equilibrium may fail to exist if the probability of the seller's low–valuation type is too high, because in this case both types of the buyer would only offer the low price. Nevertheless, equilibrium in mixed strategies can always be guaranteed.

Comparison with the joint–supply/standard bargaining model shows that welfare may well increase under some circumstances if one introduces the relatively more costly instrument of individual property–rights defence. In fact, if the ex-ante probability of the low–valuation seller is too high, then in the standard model, both types of buyers tend to make the same low offer, inpeding beneficial trades between high–valuation sellers and buyers. Nevertheless, if costs of signalling, i.e. of buying the option to say "no", are not too high, then this device will indeed guarantee that all efficient changes in property–rights do take place.

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